ICFP – Soft Matter

Rheology and shear-thickening of dense suspensions

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As the density of particles increases, the rheology of a suspension becomes more complex: its viscosity increases, and eventually diverges at the jamming volume fraction ϕ_J. The first part of this tutorial explores a one parameter model of dense suspensions, which accounts for a wealth of experimental results [1]. However, this model implies that the viscosity of the suspension does not depend on the shear rate, giving the fluid a "Newtonian" character. In particular, it does not reproduce the shear thickening allowing people to run on top of a pool filled with a dense suspension of cornstarch and water (Fig. 1). In the second part of this tutorial, we discuss a simple model for a suspension in which the contacts between the particles can be lubricated or frictional, depending on the pressure [2]. This model exhibit shear thickening and, in some range of volume fraction, discontinuous shear thickening.

1 Single parameter rheology

Using a pressure controlled rheometer, it has been shown that the rheology of a suspension is described by a single parameter, the viscous number [1]

$$I_v = \frac{\eta_f \dot{\gamma}}{P}, \tag{1}$$

where η_f is the viscosity of the suspending fluid and P the particle pressure (the pressure that is applied on the particles only through a porous plate). Then, two functions of I_v give the shear stress, $\tau = P\mu(I_v)$ and the volume fraction $\phi(I_v)$ (Fig. 2).

1. Show that this model describes a fluid with relative normal and shear viscosities, η_n and η_s, defined by $P = \eta_n(\phi)\eta_f\dot{\gamma}$ and $\tau = \eta_s(\phi)\eta_f\dot{\gamma}$.

2. The definition of the International Union of Pure and Applied Chemistry (IUPAC) for a Newtonian fluid is "*A fluid in which the components of the stress tensor are linear functions of the first spatial derivatives of the velocity components. These functions involve two material parameters (taken as constants throughout the fluid, although depending on ambient temperature and pressure)*". Is this fluid Newtonian?

3. Using the data presented in Fig. 2, show that the viscosities diverge as ϕ approaches the jamming density ϕ_m.

4. What should be the form of $\mu(I_v)$ at large I_v for the Einstein viscosity to be recovered, that is, $\eta_s(\phi) = 1 + \frac{5}{2}\phi + \mathcal{O}(\phi^2)$? In this regime, what is the particle pressure P? Explain why the particle pressure is not present in the calculation of the Einstein viscosity.

Figure 1: Snapshot of a person running on top of a pool filled with a dense suspension of cornstarch and water. From [3].

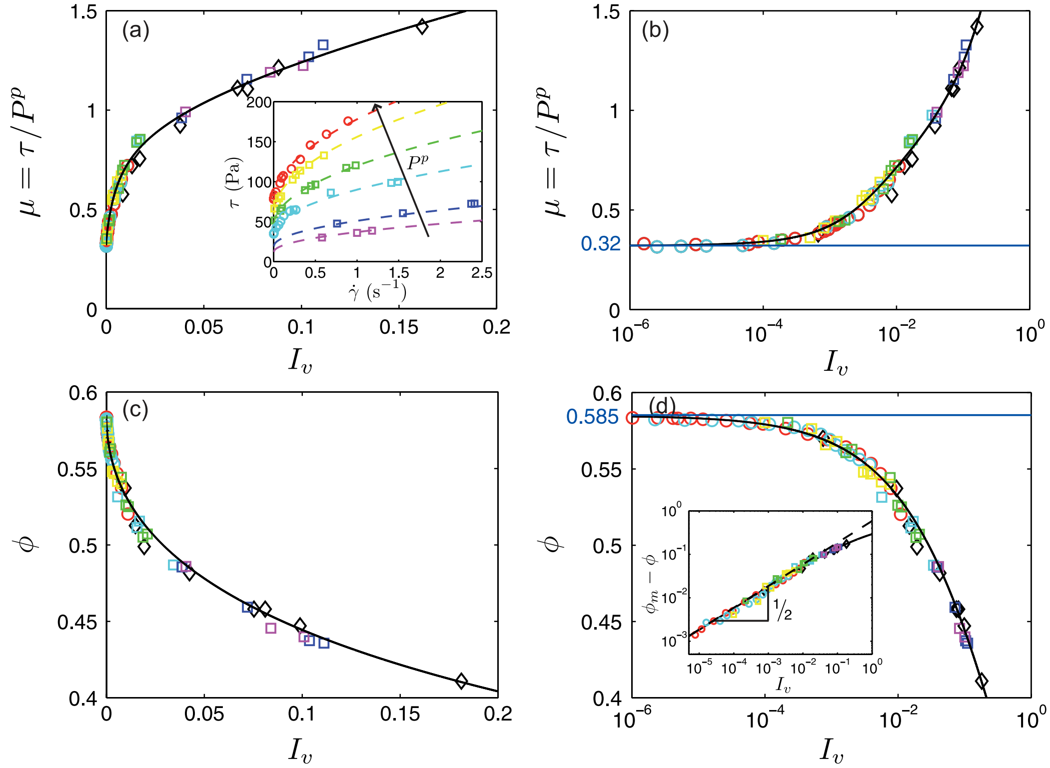

Figure 2: Friction coefficient and volume fraction as a function of the viscous number I_v. From [1]. The solid line in (c) and (d) is given by $\phi = \phi_m/(1 + I_v^{1/2})$, with $\phi_m = 0.585$.

2 Repulsive interactions and discontinous shear-thickening

Shear-thickening is absent in the model studied above, and jamming occurs at $\phi_m < \phi_0$, where $\phi_0 \simeq 0.64$ is the volume fraction at random-close packing. However, if the contacts between the particles are lubricated, jamming should only occur at ϕ_0 [4]. This means that the contacts are frictional in the experiments of Ref. [1]. In Ref. [2], it is argued that repulsive interactions can prevent frictional contacts below a pressure P^*. They thus introduce a model where the jamming density depends on the rescaled pressure $p = P/P^*$ as

$$\phi_J(p) = \phi_m + (\phi_0 - \phi_m)\mathrm{e}^{-p}. \tag{2}$$

They also assume that the pressure obeys the same kind of relation as in the previous model:

$$p = \frac{\lambda\dot{\gamma}}{[\phi_J(p) - \phi]^2} \tag{3}$$

for $\phi < \phi_J$. For $\phi > \phi_J$ the suspension is jammed and cannot flow. Moreover, they assume that the friction coefficient depends on the volume fraction but not on p.

5. * Analyse the flow curves $\dot{\gamma}(p)$ predicted by this model for volume fractions $\phi > \phi_0$, then $\phi_m < \phi < \phi_0$, and finally $\phi < \phi_m$. Show that the suspension generally shear-thickens, and that the shear-thicknening is discontinuous for a range of volume fractions.

References

[1] François Boyer, Élisabeth Guazzelli, and Olivier Pouliquen. Unifying Suspension and Granular Rheology. *Phys. Rev. Lett.*, 107(18):188301, Oct 2011.

[2] M. Wyart and M. E. Cates. Discontinuous Shear Thickening without Inertia in Dense Non-Brownian Suspensions. *Phys. Rev. Lett.*, 112(9):098302, Mar 2014.

[3] Eric Brown and Heinrich M. Jaeger. Shear thickening in concentrated suspensions: phenomenology, mechanisms and relations to jamming. *Reports on Progress in Physics*, 77(4):046602, 2014.

[4] Peter Olsson and S. Teitel. Critical scaling of shear viscosity at the jamming transition. *Phys. Rev. Lett.*, 99:178001, Oct 2007.