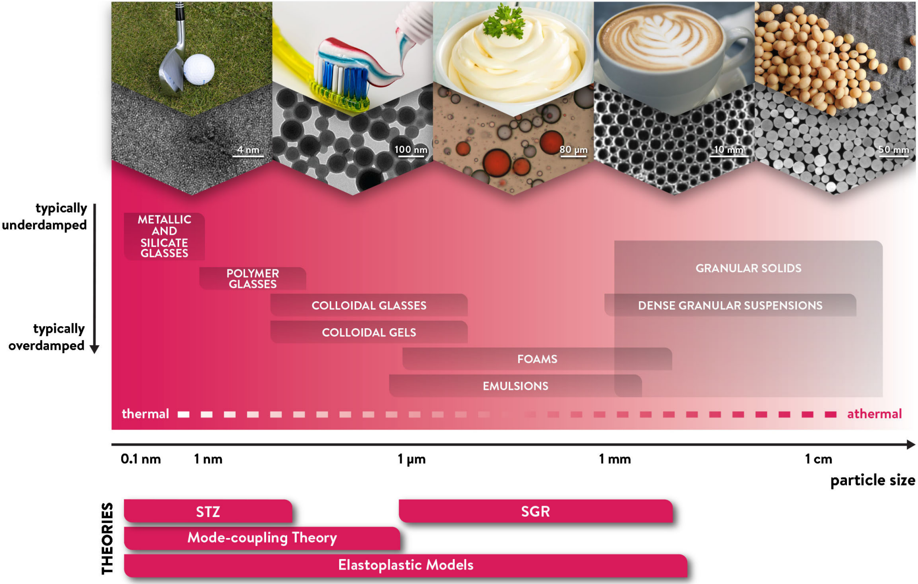
Flow of amorphous solids, elastoplastic models

[Nicolas, Ferrero, Martens, Barrat, *Rev Mod Phys* 2018]

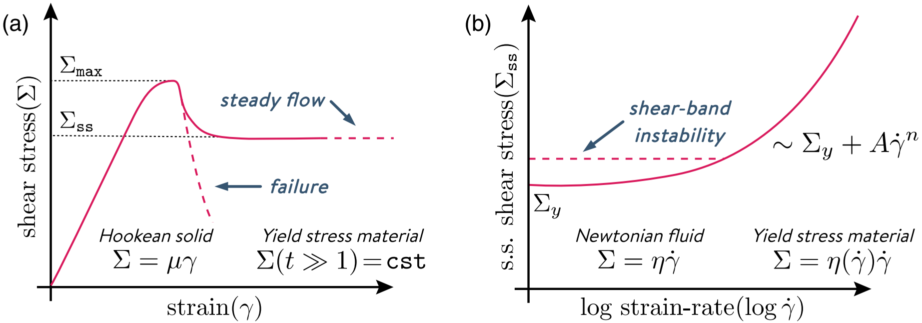
Vincent Démery

Amorphous solids



[Nicolas, Ferrero, Martens, Barrat, *Rev Mod Phys* 2018]

Macroscopic behavior



Stress-strain curve, shear transformations

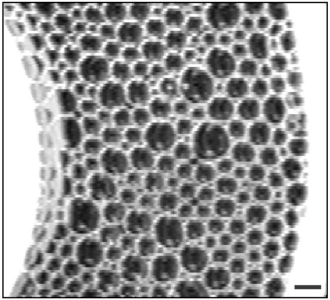


FIG. 1. This is an image of one section of a typical bubble raft. Part of both the inner and the outer cylinder is visible. The black scale bar in the lower right corner is 3.6 mm.

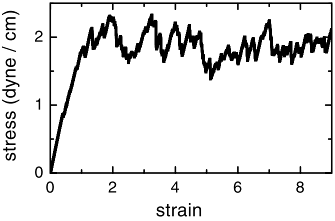
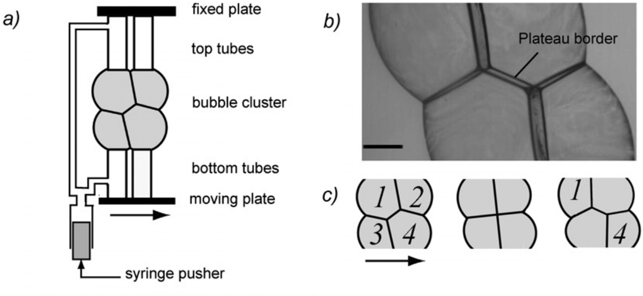


FIG. 3. Plot of the stress versus strain for a rate of strain of 3.1×10^{-3} s^{-1}.

[Lauridsen et al *PRL* 2002]

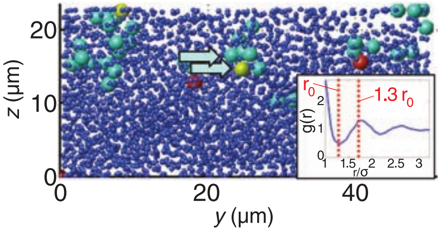
Shear transformation for bubbles (T1 event)

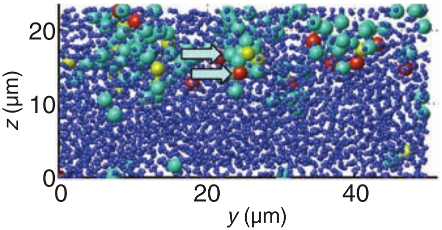


[Biance et al *Soft Mat* 2009]

Shear transformation in simulations of sheared colloidal glasses

- Slowly sheared colloidal glass.
- Big particles loose nearest neighbors (color indicates how many).





[Schall et al *Science* 2007]

Stress redistribution after a shear transformation

- Response of the system to a localized plastic strain $\epsilon^{\mathrm{pl}}(\boldsymbol{x}) = \epsilon^{\mathrm{pl}}\delta(\boldsymbol{x})$.
- Elastic and plastic contributions to the strain: $\epsilon = \epsilon^{\mathrm{el}} + \epsilon^{\mathrm{pl}}$.
- Hooke's law

$$\sigma_{ij} = 2\mu\epsilon^{\mathrm{el}}_{ij} + \lambda\epsilon^{\mathrm{el}}_{kk}\delta_{ij}.$$

- Equilibrium: $\partial_i\sigma_{ij} = 0$.
- [Calculation on the blackboard]

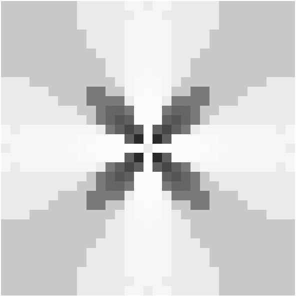
[Picard et al *EPJE* 2004]

Stress redistribution after a shear transformation: solution

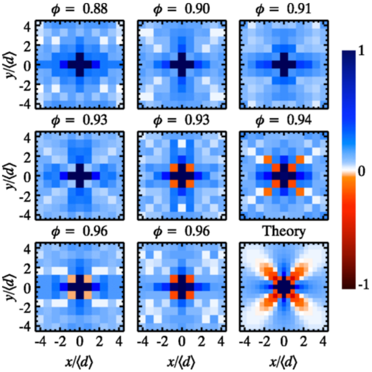
- For a shear plastic strain in dimension $d = 2$, we find

$$\sigma_{xy}(\boldsymbol{x}) \propto \frac{\cos(4\theta)}{r^2}.$$

- Measurements in an emulsion.



[Picard et al *EPJE* 2004]



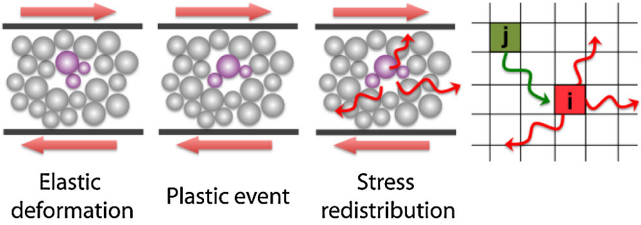
[Desmond et al *PRL* 2015]

Elastoplastic models: general structure

- Scalar model: we focus on the shear stress.
- Stress σ_i at site i.
- A site can deform elastically, $n_i = 0$, or yield, $n_i = 1$.
- The stress evolves according to

$$\dot{\sigma}_i = \mu\dot{\gamma} - |G_0| n_i \frac{\sigma_i}{\tau} + \sum_{j \neq i} G_{ij} n_j \frac{\sigma_j}{\tau}.$$

  - In the Hébraud-Lequeux model, $\tau \to 0$: the relaxation is intantaneous.
- Rules should be given for the transitions $0 \leftrightarrow 1$ for n_i.
  - In the Hébraud-Lequeux model, $0 \to 1$ with rate $\theta(|\sigma_i| - \sigma_c)/\tau_y$.

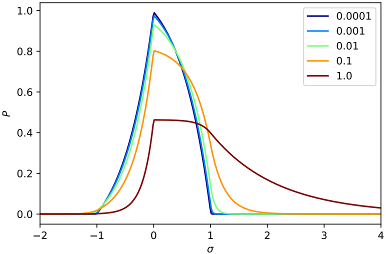


[Hébraud and Lequeux *PRL* 1998, Bocquet et al *PRL* 2009]

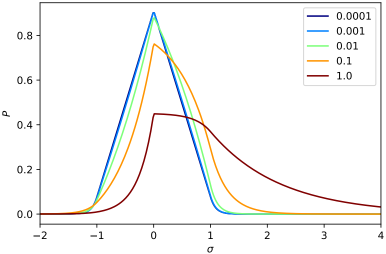
Possible refinements

- Tensorial model (redistribution, yield criterion, etc.).
- Anisotropic linear elasticity, variation of Lamé coefficients.
- Finite element resolution to reduce the effects of a square grid.

Hébraud Lequeux model: stress distribution



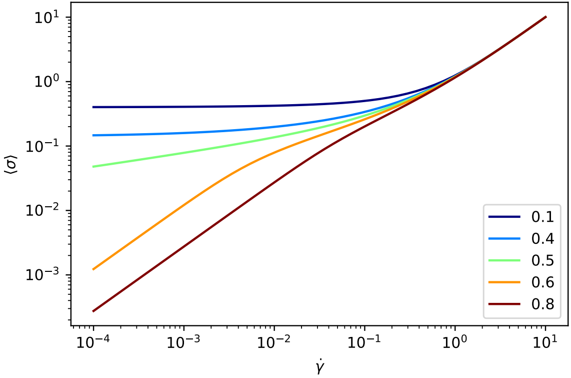
$$\frac{\alpha}{\sigma_c^2} = 0.4$$



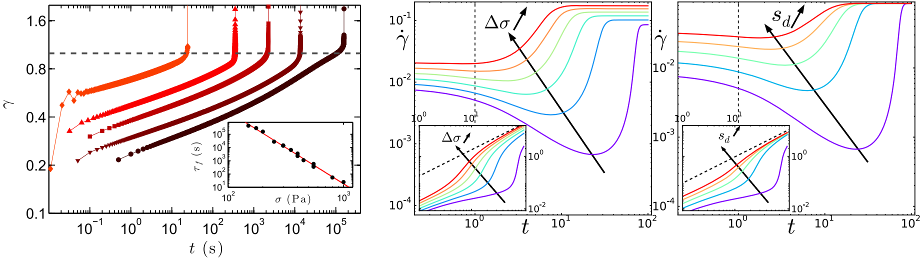
$$\frac{\alpha}{\sigma_c^2} = 0.6$$

[Hébraud and Lequeux *PRL* 1998]

Hébraud Lequeux model: stress-strain curves

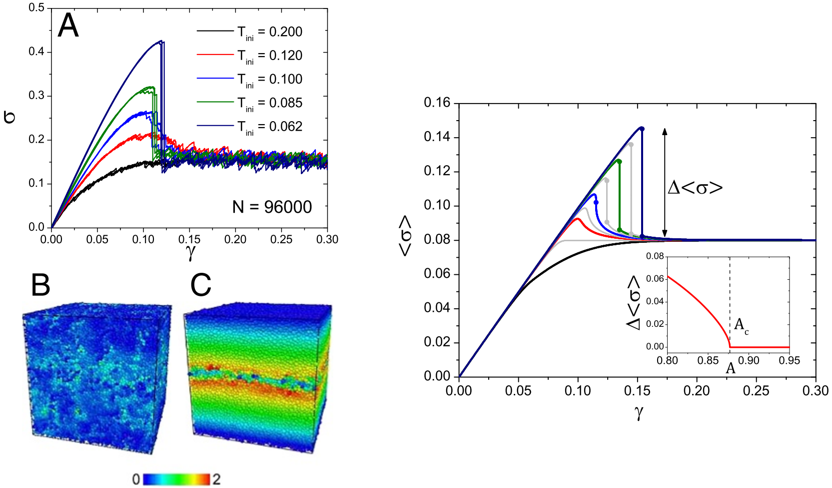


- Stress controlled driving.
- Varying stress and “age” of the gel.



[Leocmach et al *PRL* 2014, Liu et al *PRL* 2018]

Ductile to brittle yielding transition

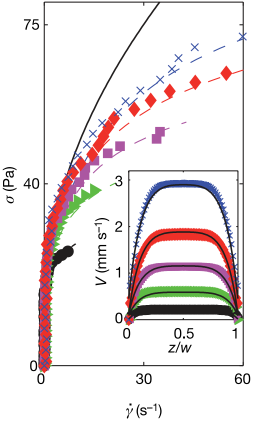


[Ozawa et al *PNAS* 2018]

Spatial dependance: kinetic elastoplastic model

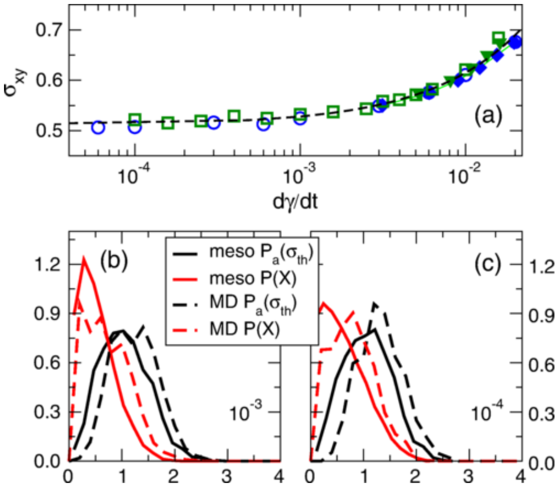
$$\partial_t P(\sigma, \boldsymbol{x}, t) = -\mu\dot\gamma(\boldsymbol{x}, t)\partial_\sigma P(\sigma, \boldsymbol{x}, t)) - \frac{\theta(|\sigma| - \sigma_c)}{\tau} P(\sigma, \boldsymbol{x}, t) + \Gamma(\boldsymbol{x}, t)\delta(\sigma) + D(\boldsymbol{x}, t)\partial_\sigma^2 P(\sigma, \boldsymbol{x}, t),$$

$$D(\boldsymbol{x}, t) = \alpha\Gamma(\boldsymbol{x}, t) + m\nabla^2\Gamma(\boldsymbol{x}, t).$$



[Goyon et al *Nature* 2008, Bocquet et al *PRL* 2009]

Parameters from microscopic models



[Puosi et al *Soft Matter* 2015, Liu et al *PRL* 2021]